RODOLFO GUIDOBONO

CURSO TEORÍA CUÁNTICA DE CAMPOS
EJERCICIO CAPÍTULO 61

JAVIER GARCÍA

Calcular $F_{\mu\nu}$ para el Grupo U(1)

Tensor de Curvatura

$$\boldsymbol{\mathcal{F}_{\mu\nu} = [\, D_\mu \,, D_\nu \,]}\ \textbf{; donde: } \boldsymbol{D_\mu = \partial_\mu - igA_\mu}$$

$$\boldsymbol{\mathcal{F}_{\mu\nu} = -ig\, F_{\mu\nu}}$$

$$\mathcal{F}_{\mu\nu}\,\psi = [\, D_\mu \,, D_\nu \,]\,\psi = (\, D_\mu\, D_\nu - D_\nu\, D_\mu)\,\psi$$

$$\mathcal{F}_{\mu\nu}\,\psi = \left(\left(\partial_\mu - igA_\mu\right)(\partial_\nu - igA_\nu) - (\partial_\nu - igA_\nu)\left(\partial_\mu - igA_\mu\right)\right)\psi$$

$$\mathcal{F}_{\mu\nu}\,\psi = \left(\partial_\mu\,\partial_\nu - \partial_\mu igA_\nu - igA_\mu\,\partial_\nu + i^2g^2A_\mu A_\nu - \partial_\nu\,\partial_\mu + \partial_\nu igA_\mu + igA_\nu\,\partial_\mu - i^2g^2A_\nu A_\mu\right)\psi$$

Como

$$A_\mu A_\nu = A_\nu A_\mu$$

$$\partial_\mu\,\partial_\nu\,\psi = \partial_\nu\,\partial_\mu\,\psi$$

Entonces

$$\mathcal{F}_{\mu\nu}\,\psi = \left(-\partial_\mu igA_\nu - igA_\mu\,\partial_\nu + \partial_\nu igA_\mu + igA_\nu\,\partial_\mu\right)\psi$$

$$\mathcal{F}_{\mu\nu}\,\psi = -ig\left(\partial_\mu A_\nu + A_\mu\,\partial_\nu - \partial_\nu A_\mu - A_\nu\,\partial_\mu\right)\psi$$

$$\mathcal{F}_{\mu\nu}\,\psi = -ig\left(\partial_\mu\,(A_\nu\,\psi) + A_\mu\,\partial_\nu\psi - \partial_\nu(A_\mu\,\psi) - A_\nu\,\partial_\mu\psi\right)$$

Considerando que

$$\partial_\mu\,(A_\nu\,\psi) = (\partial_\mu A_\nu)\,\psi + A_\nu\;\partial_\mu\,\psi$$

$$\partial_\nu\,(A_\mu\,\psi) = (\partial_\nu A_\mu)\,\psi + A_\mu\;\partial_\nu\,\psi$$

Se obtiene

$$\mathcal{F}_{\mu\nu}\,\psi = -ig\,(\,(\partial_\mu A_\nu)\psi + A_\nu\;\partial_\mu\,\psi + A_\mu\,\partial_\nu\psi - (\partial_\nu A_\mu)\psi - A_\mu\;\partial_\nu\,\psi - A_\nu\,\partial_\mu\psi)$$

$$\mathcal{F}_{\mu\nu}\,\psi = -ig\,(\,(\partial_\mu A_\nu)\psi - (\partial_\nu A_\mu)\psi)$$

$$\mathcal{F}_{\mu\nu}\,\psi = -ig\,\left((\partial_\mu A_\nu) - (\partial_\nu A_\mu)\right)\psi$$

$$\mathcal{F}_{\mu\nu} = -ig\,(\,\partial_\mu A_\nu - \partial_\nu A_\mu\,)$$

Recordando que

$$\mathcal{F}_{\mu\nu} = -ig\,F_{\mu\nu}$$

$$\boldsymbol{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$$